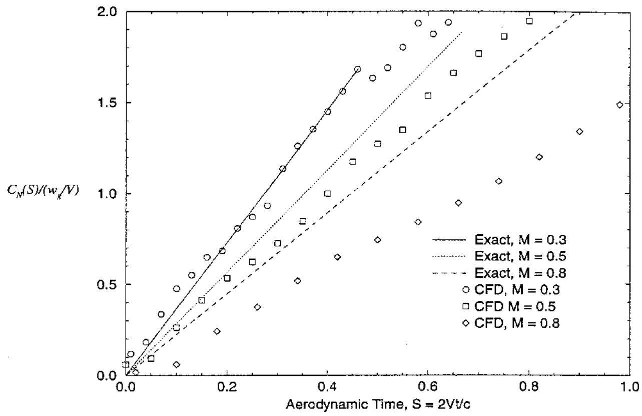

Fig. 3 Comparison of CFD with exact results for gust response at small times.

where w_g is the gust vertical velocity. The comparison is excellent for $M_\infty = 0.3$, whereas the agreement becomes progressively worse for $M_\infty = 0.8$.

Recently, this same method has been extended to be three-dimensional to examine the indicial response of wings to a step change in angle of attack and the resulting detailed flowfield development.[7]

Conclusions

A method has been developed to calculate the indicial and gust responses of an airfoil in compressible flow directly using CFD. Previous work with Euler/Navier–Stokes codes have been restricted in the sense that the indicial responses can be calculated only indirectly using the Laplace transform approach. Furthermore, such methods cannot give the surface pressure time histories, and hence, may not be useful in gaining insight into the physical features of the flow. The present method, using a grid-velocity approach, has been demonstrated to be accurate via comparison with known exact analytical results at $t = 0$ and $t = \infty$ and via comparison with exact linear theory results for small times. The comparison is excellent for $M_\infty = 0.3$, whereas the agreement becomes progressively worse as the flowfield becomes increasingly nonlinear.

Acknowledgments

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Flutter Analysis Using Unsteady Aerodynamics in Non-Pade Form

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Introduction

THE linearized equations of motion (EOM) of a flexible aircraft contain unsteady aerodynamic terms that depend on the Mach number M and the reduced frequency k. The exact dependence of the aerodynamic coefficients on M and k cannot be expressed in the form of algebraic functions. As a result, these coefficients are computed for each desired Mach number for a set of predetermined values of reduced frequencies. These values will be referred to as the tabular values of the complex aerodynamic coefficients. The classical methods of solution[1–3] of the flutter equations involve the k, p, and $p-k$ methods (with the Pade method[4–6] considered as a form of the p method). The k method leads to complex EOM, which are solved using the tabular values of the aerodynamic coefficients obtained for simple harmonic oscillations. Therefore, the complex eigenvalues obtained before and after the flutter point do not reflect the damping of the different structural modes. The p and the $p-k$ methods yield complex eigenvalues that attempt to represent the actual damping of the structural modes. The $p-k$ method yields EOM that may assume either complex coefficients[1] or real coefficients,[3] and it is based on the tabular values of the aerodynamic coefficients (or on interpolated values, for values of k between adjacent tabular points), without attempting to fit them into any explicit functional form. The EOM in the $p-k$ method are solved in an iterative fashion so that the assumed value of k (for which oscillatory aerodynamic coefficients are either available as tabular values, or as interpolated values), converges to the computed value of the imaginary part of a preselected eigenvalue (at a chosen airspeed). The iterations are repeated, for a single mode at a time, until all of the modes are treated for convergence in the manner just described. The p method avoids the previous iteration process by using explicit expressions that approximate the transient aerodynamics in the time domain. The main difficulty with the p method lies in the derivation of the approximating expressions in the time domain for configurations that employ lifting surface aerodynamics. The Pade method, which is essentially a p method, is based on using the tabular values of the complex aerodynamic coefficients, derived for oscillatory motion, for the purpose of fitting them into an explicit expression involving rational polynomials that are eventually used in the Laplace domain. The most common Pade representation[4] assumes the following form:

$$A(k) = A_0 + A_1 ik + A_2(ik)^2 + \sum_{j=1}^{n_L} \frac{A_{2+j} ik}{(ik + \beta_j)} \tag{1}$$

where all of the A_j are real matrices, β_j is a real lag term, and $i = \sqrt{-1}$. The Pade representation enables one to cast the aeroelastic EOM into a set of first-order differential equations with constant coefficients, a form that enables the use of optimal control theory for the design of control laws to suppress flutter and/or improve the aeroelastic behavior of the structure. However, the introduction of the rational terms, referred to also

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as the lag terms, into Eq. (1) leads to a dramatic increase in the order of the EOM. For example, a system with n_s structural modes will be augmented by n_s states for each rational term added to Eq. (1). To obtain a good accuracy using Eq. (1), as many as four lag terms may be required, thus leading to $4 \times n_s$ augmented states. The minimum state (MS) method[6] uses a Pade approximation of different form to yield a smaller number of augmented states. A typical aeroservoelastic problem, using the MS method with physical weighting (its most efficient form), may require around eight augmented states. However, the MS method requires extensive double least-square-type iterations for convergence. Unlike the k method, all of the other methods (that is, the p method, including the Pade method, and the $p-k$ method) give good indications regarding the rate of decay of the structural modes, provided this rate of decay assumes small values. In the present work an attempt is made to approximate the aerodynamic coefficients into an explicit expression in a manner similar to the one used in the Pade approximation, but in such a way as to completely avoid the penalty of augmented states.

Proposed Method

A study of Eq. (1) reveals that since $A(k)$ is complex and the A_j are real, the real part of $A(k)$ is approximated by the parabola involving $A_0 - A_2k^2$ only, and the imaginary part is approximated by the linear function A_1k only. This is the reason why additional rational terms are needed in Eq. (1) to improve the previous polynomial representation. Another way is to allow both the real and imaginary parts of $A(k)$ to be approximated by a three-term parabola in ik, thus leading to a complex coefficient polynomial representation of the form

$$A(k) = A_{0R} + iA_{0I} + (A_{1R} + iA_{1I})ik + (A_{2R} + iA_{2I})(ik)^2 \quad (2)$$

Equation (2) results in complex EOM and it altogether avoids any augmented states. It will be shown that Eq. (2) may be made to represent an adequate approximation of $A(k)$ over a range of k, and therefore it can be used for all aeroelastic and aeroservoelastic applications, with the exception of optimal control law design only. It may readily be applied, for instance, to flutter calculations that involve a given control law, or design a control law using parametric optimization of selected variables, or perform routine flutter calculations. It is the purpose of this work to test the accuracy of the approximation presented in Eq. (2), together with its validity to flutter analysis.

Accuracy of the Aerodynamic Approximation

The accuracy of the previous aerodynamic approximation is tested in two ways:

1) The first is by direct comparison between the tabulated aerodynamic coefficients with those interpolated using Eq. (2).

2) The second is by applying the approximation to a flutter example using both open- and closed-loop analysis. It may be stated right at the outset that the approximation represented by Eq. (2) is not satisfactory when trying to fit it over a wide range of reduced frequencies. It is customary to fit all of the aerodynamic coefficients over the whole range of reduced frequencies for which the tabulated data are available. However, inspection of a typical root locus plot, such as the one shown in Fig. 1, indicates that the variation with dynamic pressure Q_D of the frequency of each structural mode is very limited. Therefore, for any assumed velocity, there is no need to fit all of the aerodynamic coefficients over the full range of the tabular k values, but only over a more limited range of reduced frequencies that spans only the neighboring structural modes. Thus, for the first mode, we do not need to use k values that exceed those of the third or fourth modes. The rule that we adopt in this work is to choose a range of k, for each mode, that spans two structural modes with lower frequencies and two structural modes with higher frequencies. Thus, for the ith mode, all of the j terms corresponding to the ith equation (that is all of the A_{ij} terms) will be fitted over the same selected range of k described earlier. For the first two modes having less than two lower structural modes, a correspondingly larger number of modes with higher frequencies is taken. Similarly, for the two highest modes, a correspondingly larger number

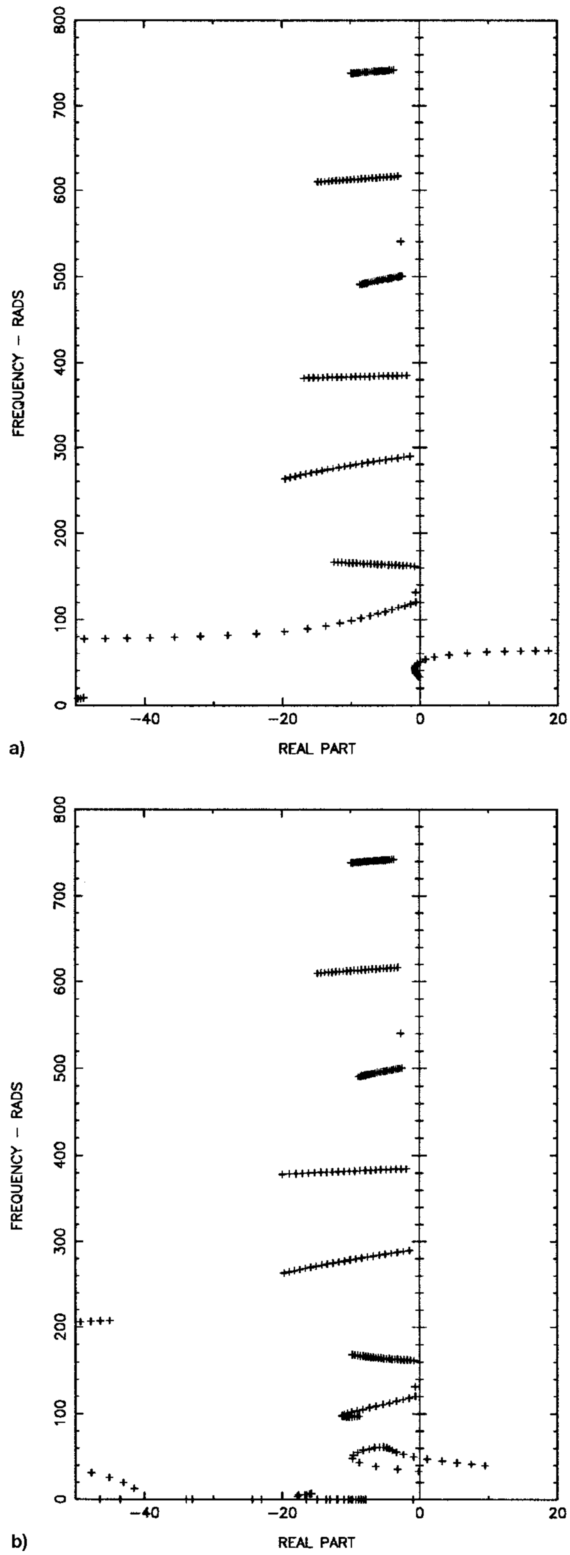

Fig. 1 Root-locus plots—DAST–ARW1, using Pade approximation with four lag terms leading to 40 augmented states, M = 0.9: a) open-loop case, Q_F = 99.7 psf and ω_F = 50.2 rad/s and b) closed-loop case, Q_F = 174.4 psf and ω_F = 48.8 rad/s.

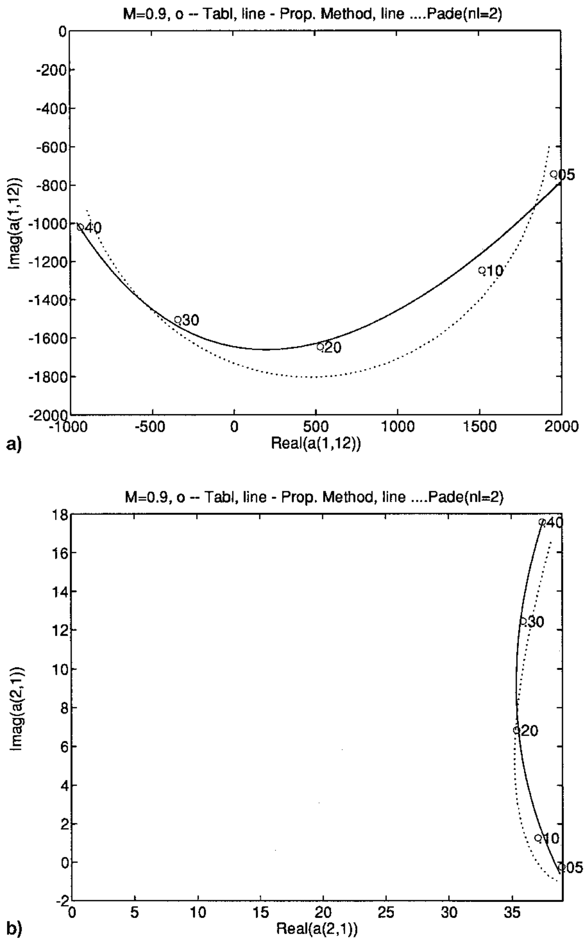

Fig. 2 Variation of aerodynamic coefficients with k: a) variation of $a(1,12)$ and b) $a(2,1)$.

of lower modes is taken so that the selected range of k spans between four and five modes. If rigid body modes (or static divergence) are considered, one may readily force $A_{0I} = 0$ for these specific modes to avoid possible static anomalies. Figure 2 presents an example of aerodynamic coefficients fitted using Eq. (2). These coefficients relate to the DAST–ARW1[7] mathematical model, which consists of 10 structural modes, 1 control mode (mode 11), and 1 gust mode (mode 12), at $M = 0.9$. The circles represent the tabular data, with the value of k marked next to each circle, whereas the solid line represents the values computed using Eq. (2), with the coefficients determined by the normal least-square fit, using values of k that span a wider range than the one required by the aforementioned rule. This conservative presentation is done to show the quality of the approximation represented by Eq. (2). The dotted line represents the values obtained using Pade approximation with two lag terms [see Eq. (1)]. As can be seen, the fit using Eq. (2) appears to be very good and is better than the one obtained using Pade with two lag terms. This was found to be true for almost all of the coefficients, although their associated figures are not included herein. At this stage there remains to be shown that the previous restricted range of k over which the aerodynamic coefficients are fitted does not adversely affect the computed flutter values.

Results Involving Flutter Calculations

For the sake of completeness, the following EOM, based on Eq. (2), are presented:

$$\{[M_s + \tfrac{1}{2}\rho b^2(A_{2R} + iA_{2I})]p^2 + [B + \tfrac{1}{2}\rho bV(A_{1R} + iA_{1I})]p + [K + \tfrac{1}{2}\rho V^2(A_{0R} + iA_{0I})]\}q = 0 \tag{3}$$

where $p = \mu + i\omega$ replaces the $i\omega$ terms in the ik in Eq. (2), M_s, B, and K are the generalized mass, damping, and stiffness matrices, respectively. V represents the velocity, ρ the density, and b the reference length used in determining k. Following all of the flutter methods, consideration should be given only to those values of p with positive frequencies (that is, with positive imaginary values). Equation (3) requires that values for ρ, V, and M be assumed, and the solution is then sought for the eigenvalues p using routines for finding eigenvalues of complex matrices. One can match ρ, V, and M for particular flight conditions, or alternatively, change ρ (or the dynamic pressure Q_D since $\rho = 2Q_D/V^2$), as indicated in Ref. 7. It should be stressed here that, unlike the p–k method, Eq. (3) requires no iterations.

For comparison purposes, flutter calculations are made using the Pade approximation with four lag terms. This Pade approximation was tested extensively in many publications and found to be very satisfactory.[8] However, it causes the 10-mode flutter example to be augmented to yield flutter equations of order 60 × 60 for the open-loop case, and 71 × 71 for the closed-loop case. The actuator dynamics together with the control law that was designed for this model in Ref. 7 (using the outboard wing sensor) are used herein. The root–locus plots for these two cases are presented in Fig. 1, where Q_D is varied between $Q_D = 0$–220 psf, in increments of 10 psf. As can be seen, the calculated open-loop flutter dynamic pressure is $Q_F = 99.7$ psf with flutter frequency $\omega_F = 50.2$ rad/s, and the computed closed-loop flutter dynamic pressure is given by $Q_F = 174.5$ psf with $\omega_F = 48.7$ rad/s. The previous values will be referred to as being the exact values, only for purposes of comparison with the approximate fit introduced in this work. The respective results for the open- and closed-loop cases using Eq. (2) are not shown here, since they are essentially identical to Fig. 1 for all modes other than those associated with the augmented aerodynamic states. The k range used for the different modes is given in Table 1. The different flutter results obtained are given in Table 2. As can be seen, the proposed method yields open-loop flutter given by $Q_F = 97.9$ psf and $\omega_F = 49.7$ rad/s, and closed-loop flutter given by $Q_F = 175.7$ psf with $\omega_F = 48.5$ rad/s. It can thus be seen that very good flutter results are obtained when using Eq. (2). To show the importance of fitting the aerodynamic coefficients with restricted k range, flutter calculations are repeated while fitting Eq. (2) over the full k range. The results are presented in Table 2. As can be seen, these results are clearly not as good as those obtained using the aerodynamic fit with the restricted range, but they give an idea regarding the sensitivity of the results to the width of the chosen k range. It can be concluded that the restricted range is important for getting accurate results and that the results are not overly sensitive to this range so as to be extremely dependent on its values. For comparison purposes, flutter results were also computed using Eq. (2) with real coefficients, instead of complex coefficients [or alternatively, using Eq. (1) with no lag terms], while using the same restricted k range as the one used for the complex coefficients (see Tables 1 and 2). These latter results are presented in Tables 1 and 2 and show the importance of combining the restricted k range together with the complex coefficients in Eq. (2). At this stage it is important to state that the order of Eq. (3) is 20 × 20 for the open-loop case (compared with 60 × 60 using the exact model) and 31 × 31 for the closed-loop case (compared with 71 × 71 using the exact model).

Discussion

It can rightfully be argued that since Eq. (2) leads to complex EOM, the computational labor involved is twice as large as the one involved with the same order of equations having real coefficients. Therefore, for the numerical example presented earlier, a 20 × 20 complex eigenvalue problem corresponding to the open-loop case using Eq. (2) involves just

Table 1 Range of reduced frequencies used in flutter calculations for fitting the aerodynamic coefficients of the different modes

Mode	1	2	3	4	5	6	7	8	9	10
k_{min}	0.05	0.05	0.05	0.05	0.10	0.10	0.20	0.30	0.40	0.50
k_{max}	0.30	0.30	0.30	0.30	0.40	0.50	0.60	0.70	0.80	0.80

Table 2 Summary of the different flutter results

Flutter case	Exact results Pade with $n_L = 4$	Proposed method restricted k range	Proposed method unrestricted k range	Proposed method real coefficients restricted k range	Pade with $n_L = 1$	Pade with $n_L = 2$	Pade with $n_L = 3$
Open loop							
Q_F, psf	99.7	97.9	110.5	120.2	96.8	99.1	98.9
ω_F, rad/s	50.2	49.7	53.8	57.8	49.3	49.9	50.0
Closed loop							
Q_F, psf	174.5	175.7	171.6	166.6	181.3	172.2	173.5
ω_F, rad/s	48.7	48.5	50.9	53.4	46.4	48.9	49.3

about the same computational labor as a 30 × 30 eigenvalue problem having real coefficients obtained using the Pade representation with a single lag term. Hence, the method proposed herein can be justified in terms of computational efficiency only if it proves to have some advantages over the Pade representation with small number of lag terms. However, as stated earlier, the study of figures similar to Fig. 2 have shown that the proposed approximation yields an accuracy that is higher than the one obtained using Eq. (1) with two lag terms. Table 2 summarizes the flutter results obtained with various n_L lag terms. It can be seen that the results obtained using Pade representation with $n_L = 3$ are only very slightly superior to those obtained using the complex polynomial representation given by Eq. (2). Hence, even if we assume that the proposed method yields results comparable only to the Pade representation with $n_L = 2$ (a somewhat conservative assumption), then the open-loop case turns to be twice as fast compared to the Pade method (with $n_L = 2$). Lastly, it can be argued that good flutter results can be obtained using a smaller number of lag terms, when using the MS method. This is all true. However, the MS method requires an iterative double least-square method for fitting the aerodynamic coefficients, which needs to be taken into account when considering its relatively small number of lag terms.

Conclusions

In conclusion it can be stated that a method is presented by which a p-type flutter analysis can be performed using complex EOM with absolutely no lag terms and no iterations. It is shown that the combination of complex coefficients, together with a restricted range of k over which these coefficients are fitted, is responsible for the high accuracy obtained. This proposed method can replace the $p-k$ method and may readily be used to perform routine open-/closed-loop flutter calculations, or to design control laws using parametric optimization of selected variables. However, it cannot be used, in its present form, to design control laws using optimal control theory, since the resulting equations are complex.

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Cascade Optimization Strategy for Aircraft and Air-Breathing Propulsion System Concepts

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Introduction

DESIGN optimization for aircraft and air-breathing propulsion engine concepts has been accomplished by soft-coupling the flight optimization system (FLOPS)[1] and the

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